

Discovering Properties of Logarithms

There are certain properties of logarithms that are very important in applying mathematics to real world problems. All of these properties are based on the concept that “the logarithm of a number is an exponent”. We already know two of these properties of logarithms.

1. $\log_b 1 =$ _____

2. $\log_b b =$ _____

I. With the help of our graphing calculator, we will try to discover some other properties of logarithms. Written below are equations involving logarithms that **may** or **may not** be true. With your graphing calculator, put the expression on the left side of the equation into Y1 of your calculator, and the expression on the right side of the equation into Y2. From the graphs of each side of the equation decide whether you think the equation is **TRUE** (for all values of x) or **FALSE**. Set the WINDOW of your calculator so you have a viewing rectangle of:

Xmin:-3 Xmax:10

Ymin: -2 Ymax: 3

1. $\log(3 + x) = \log 3 + \log x$ **TRUE** or **FALSE**

2. $\log(3 \cdot x) = \log 3 + \log x$ **TRUE** or **FALSE**

3. $\log(x^3) = (\log x)^3$ **TRUE** or **FALSE**

4. $\log(x^3) = 3(\log x)$ **TRUE** or **FALSE**

5. $(\log x)^3 = 3(\log x)$ **TRUE** or **FALSE**

6. $\log(x/4) = \log x / \log 4$ **TRUE** or **FALSE**

7. $\log(x - 4) = \log x - \log 4$ **TRUE** or **FALSE**

8. $\log(x/4) = \log x - \log 4$ **TRUE** or **FALSE**

9. $\log x / \log 4 = \log(x/4)$ **TRUE** or **FALSE**

II. Look carefully at the statements above that you said were **TRUE**. Find patterns that allow you to write some **general rules** for logarithms.

1. $\log(a \cdot b) =$ _____

2. $\log(a/b) =$ _____

3. $\log(a^n) =$ _____

III. In applying properties of logarithms, it is very easy to write an equation that is not true. There is an error in each of the following equations that make them **NOT** true. Look carefully at the statements in part I that you said were **FALSE** and the properties that you wrote in part II. Find the error in each statement below and rewrite one side of equation to make it **TRUE**.

1. $\log(a + b) = \log a + \log b$ _____

2. $n(\log a) = (\log a)^n$ _____

3. $\log a / \log b = \log(a / b)$ _____

IV. Use the properties of logarithms (in part II) to rewrite the following expressions in terms of $\log a$, $\log b$, and $\log c$.

Example: $\log\left(\frac{a \cdot \sqrt[3]{b}}{c^4}\right) =$

1. $\log(a^2 b^3) =$

2. $\log\left(\frac{a}{\sqrt{c}}\right) =$

3. $\log\sqrt{\frac{c}{b}} =$

4. $\log\left(\frac{a^3}{bc^2}\right) =$

V. There are two more important properties of logarithms. These are results of the fact that “a logarithmic function is the inverse of an exponential function”. Graph the following expressions and, from the graph, complete the statement.

1. $\log 10^x =$ _____

2. $10^{\log x} =$ _____